## - glossary SYSTEMS ANALYSIS

## and Planning -

 Programming - Budgeting
U. S. GENERAL ACCOUNTING OFFICE
"With a high degree of confidence we can say that the intuitive solution to problems of complex social systems will be wrong most of the time."

Jay W. Forrester<br>Massachusetts Institute of Technology

This glossary related to systems analysis and to planning-programming-budgeting (PPB) was prepared for use by professional staff members of the United States General Accounting Office (GAO). It is a revision of a similar glossary dated January 1968.

Some of the definitions of terms in the earlier glossary have been revised in an attempt to clarify them and to make them less technical, and new terms have been added to make the glossary more useful in the GAO systems analysis and statistical sampling training courses. Because of the nontechnical nature of the definitions in this booklet users who require detailed technical information concerning the terms are referred to the growing number of books which relate to the subjects involved.

Suggestions and comments on the glossary are welcomed. If they are made in writing, they should be addressed to the Director, Office of Policy and Special Studies, U.S. General Accounting Office, 441 G. Street N.W., Washington, D.C. 20548

The Role of Systems Analysis and PPB in Government

Systems analysis has been defined in many ways and in many contexts. Different definitions result from the use of the term by different specialists to define their areas of interest. In all the definitions, however, the aim of systems analysis is to select or design the best combinations of elements, components, relationships or procedures, or all of these, to achieve some objective.

In this glossary, systems analysis is described in the context of decisionmaking by Government executives. In this context the term encompasses the entire analytical process which should take place in order that decisionmakers can make informed judgments on major questions of choice--choices which are reflected in plans, programs and budgets. Thus the term places major emphasis on the search for alternative ways of achieving objectives, and its "system" orientation perceives of a program or activity as requiring inputs and producing outputs. Because of these characteristics systems analysis fits into and indeed is a necessary requirement of a successful PPB system and of efficient and effective management in Government.

The PPB system represents an effort by the executive branch of the Federal Government to prepare budgets in such a way as to make them most useful in establishing priorities, in planning, in choosing among alternative programs, and in measuring costs against performance yardsticks. A PPB system was started by the Department of Defense in 1961 and PPB was prescribed by the President in August 1965 for adoption by all major Federal agencies.

An expression of GAO opinion regarding the issues raised by the application of systems analysis and PPB in the executive branch and the related implications for the Congressional role in establishing national policies and budgets was contained in the Comptroller General's statement before the Subcommittee on National Security and International Operations of the Committee on Government Operations, United States Senate, on March 26, 1968. The planning-programming-budgeting system was described by the Comptroller General as an effort to establish on a Government-wide basis a common approach and procedure for:

1. Establishing longer range planning in terms of Federal objectives and goals as defined by the Congress or the President.
2. Identifying the most advantageous programs to fulfill these objectives on the basis of an analysis of costs and benefits of available alternatives.
3. Translating programs into budgetary and legislative proposals and longer term projections。

In June 1969 the Director of the Bureau of the Budget commented on the need for PPB and some of the difficulties expected in its implementation:
"*** I firmly support the objectives of the PPB System. Indeed, it is very difficult to see how anyone could be opposed to the objectives in that system. No one can seriously contend that we do not want to know what our programs are doing now or will do in the future under present or alternative policies. No one would suggest that we adopt a know-nothing attitude toward the use of organized knowledge.
"While this Administration has a commitment to bring all possible resources to bear in solving the Nation's problems, we are fully aware that improving and increasing program evaluation in Government is not an easy job. This is the case because Government pursues multiple objectives, provides goods and services that do not have the benefit of valuation in the market economy, and requires the participation of a multitude of decisionmakers--including, ultimately, the public itself. Nonetheless, we are making a concerted effort to improve and increase program evaluation."

In view of the growing use of systems analysis in Government, GAO professional staff members may occasionally encounter some terms with which they are not familiar. It is hoped that this glossary will prove useful by providing at least a limited understanding of such terminology.

# GLOSSARY <br> for <br> SYSTEMS ANALYSIS <br> and <br> PLANNING-PROGRAMMING-BUDGETING 

## Accrual Accounting

An accounting system in which revenues and expenditures are recognized as they are earned or occur regardless of when payment is made or when the income is actually received. An accrual accounting system reflects the resources available to an agency, the receipt of goods and services, the use of resources in relation to work performed and benefits derived during a particular time period, and the liabilities of the agency. For management it enables more effective controls because it provides data on all available resources and on expenses that can be compared with and related to program performance during a given period. Accrual accounting in Federal agencies is required by P.L. 863 (August 1, 1956). Frequently it is contrasted with the cash basis of accounting which emphasizes cash receipts and disbursements during a given period.

## Accrued Expenditures

The charges incurred during a specific period requiring the provision of funds for (1) goods and other tangible property received (2) services performed by employees, contractors, grantees, lessors and other payees, and (3) amounts becoming owed under programs for which no current service or performance is required (such as annuities, insurance claims, other benefit payments, and a few cash grants). Expenditures accrue when work is performed or resources delivered regardless of when payment is made or when resources are used.

The portion of any such expenditures which is unpaid at a given point in time is a liability. The
portion of payments made for which the expenditures have not accrued (such as advances and prepayments) is an asset. The Budget Commission recommended in 1967 that the Federal Budget show accrued expenditures in lieu of disbursements. The Commission expressed the belief that once "expenditures" have been redefined for budget purposes to fit the accrued expenditures concept, there will be no need to use the term "accrued expenditures," and the term "expenditures" will automatically apply to the budget figures developed on the accrual concept.

## Activity

A term which is often used to refer to kinds of work, of which various mixtures are required in carrying out a program (p. 52). For example, research and development, distribution of information, and training of personnel may be activities applicable to a particular agency program.

## Administrative Budget

The most familiar of three types of Federal budgets used prior to fiscal year 1969. It excluded expenditures from trust funds such as social security benefits and grants to states for highway construction, and certain receipts such as social security contributions and excise taxes which were earmarked for trust funds. The budget surplus or deficit in the administrative budget was on a cash basis except for interest expense which was on the accrual basis. Expenditures and receipts of government enterprises, such as the Post Office, were shown only if wholly owned by the Federal Government and were shown net, that is, difference between expenditures and receipts. See also: Budget.

A procedure the purpose of which is to present a convincing comparison of the relevant alternatives being considered in an analysis. The procedure is to handicap the apparently preferable alternative by making assumptions designed to place this alternative at a disadvantage as compared with the other alternatives. See also: Sensitivity analysis.

## Algorithm

A set of ordered procedures, steps, or rules, usually applied to mathematical procedures, and assumed to lead to the solution of a problem in a finite number of steps.

## Allotment Procedure

A procedure whereby an agency head or other authorized agency employee distributes the agency's apportioned funds to those authorized to incur obligations. See also: Apportionments.

## Alternatives

Within any one agency, this term means other possible programs besides those already decided upon. It denotes a comparison of two or more programs (that is, two or more possible approaches) as possible ways of fulfilling the same objective. Used in this context the term is output-oriented; it suggests substituting an entirely different program (and therefore a different output or outputs) for a program already planned or in process. On the other hand, alternative ways to do a job which has been decided upon takes the program as given, and raises possibilities for changing the mix of inputs.

## Analog Method of Cost Estimating

A method of cost estimating which is based on historical costs that are too limited to allow statistical
estimating and which is more economical to prepare than an engineering estimate. Graphical analysis is often helpful to more clearly understand the degree of relationship of the data points. The analog estimate is normally prepared by adjusting the historical cost of a similar (analog) item by deducting historical costs of components which are not comparable and adding estimated costs of new components. See also: Statistical method of cost estimating; Cost estimating relationship; Engineering method of cost estimating.

## Analysis of Variance

A procedure for making a statistical inference (p. 65) about differences among the means (p. 37) of two or more groups. The question under analysis is whether differences among the means are greater than the variability which can be expected to result from random differences between elements within the groups. The procedure involves a statistical test of the hypothesis that the samples were drawn from the same universe.

## Apportionment

A distribution made by the Bureau of the Budget of amounts available for obligation and expenditure in an appropriation or fund account into amounts available for specified time periods, activities, functions, projects, and objects, or combinations thereof. The amounts so apportioned limit the obligations to be incurred or, when so specified, expenditures to be accrued.

## Appropriation

Ordinary current appropriations (either no-year or one or more years) are budget authorizations granted currently by Congress, both to incur obligations and to make expenditures in a definite specified amount. Has excluded contract authorizations and
authorizations to spend debt receipts. Under the Budget Commission's recommendations, these latter types of authorizations are also called appropriations, but appropriations to liquidate contract authorizations would not be counted as new appropriations.

Arithmetic Mean

See: Mean.

## Asymptote

In terms of the graph of a function (p. 26) an asymptote is a straight line which the graph continually approaches and with which it coincides only at an infinite distance. It represents a boundary or limit which the function never crosses.

## Attribute

A quality. Sampling for attributes is sampling where each unit is found either to have or to lack some characteristic. This is contrasted with sampling for variables, where a numeric measurement is recorded for each item. In sampling for attributes the objective is to estimate the number of times a characteristic occurs in a population, often expressed as a percentage of the total.

Authorization

An Act of Congress which authorizes Federal programs, obligations, or expenditures. The term sometimes refers to basic substantive legislation setting up a program or an agency, and authorizing appropriations to be made for them, but not actually providing authority to spend. See also: Appropriation.

A school of thought within statistics in which estimates of probabilities of events are based on the scientist's or decisionmaker's subjective beliefs as modified by empirical data. In classical statistics probability estimates are based solely on objective data. A consequence of this difference is that Bayesian statistics is considered more decisionoriented than classical statistics since the point of "enough information" for a decision is reached more quickly under Bayesian statistics. An additional aspect of the Bayesian approach which makes it more decision-oriented is that it explicitly takes into account the cost of obtaining additional data.

Benefit-Cost Analysis
See: Cost-Benefit Analysis.

## Benefit-Cost Ratio

An economic indicator of efficiency, computed by dividing benefits by costs. Usually, both the benefit stream and the cost stream are discounted so that the ratio reflects efficiency in terms of the present value (p. 49) of future benefits and costs.

Bias

The existence of a factor which causes the estimate being made on the basis of a sample (p. 59) to differ systematically from the population measure being estimated. It may originate from poor design of the sample, from deficiencies in carrying out the sampling process, or from an inherent characteristic of the estimating technique used. Often the degree of bias related to an estimating technique is so small as to be of no practical importance. For example, the use of ratios is not expected to furnish unbiased estimates, however, the related bias is so small that in view of desirable features related to their use ratios are often used.

A probability distribution (p. 51) of the discrete type which is used for predictions related to variables or phenomena which can assume one of only two possible values, for example, success or failure, correct or incorrect, defective or nondefective. It provides a means of predicting the number of results of one kind or the other to be obtained in a given number of trials, experiments or inspections. An example of its use would be in determining the probability of obtaining 2 sixes in 10 rolls of a die.

## Boolean Algebra

A process of reasoning, or a deductive system of theorems using a symbolic logic, and dealing with classes and propositions. It employs symbols to represent operators such as AND, OR, NOT, EXCEPT, IF. . . .THEN, to permit mathematical calculation.

## Budget

A financial plan serving as a pattern for and control over future operations; hence, any estimate of future costs; any systematic plan for the utilization of manpower, material, or other resources. The term "budget" in the Federal budget context also refers to the summary totals of appropriation, receipts, expenditures (excluding net lending), expenditure account surplus or deficit, gross and net lending, total expenditures, and total budget surplus or deficit.

Budgeting
Budgeting is the process of translating planning and programming decisions into specific projected financial plans for relatively short periods of time. Budgets are short-range segments of adopted action
programs which set out planned accomplishments and estimate the resources to be applied for the budget periods in order to attain those accomplishments.

## Central Limit Theorem

If independent random samples (p. 59) are drawn with replacement from a sufficiently large population (p. 49) and the mean (p. 37) of each sample is used to form a distribution of sample means, this distribution will be approximately normal with a mean approximately equal to that of the true population and a standard deviation (p. 64) equal to the true standard deviation divided by the square root of the number of sample means. This theorem is important because it means that the normal distribution (p. 41) (bell-shaped curve) can be used in statistical inference (p. 65) to make estimates regarding a population on the basis of information obtained from a sample.

## Coefficient

A number written before a quantity to indicate multiplication, that is how many times the quantity is to be taken additively. For example, in the expression 5 ax the coefficient of the quantity ax is 5 while the coefficient of the quantity $x$ is 5 a.

Coefficient of Correlation

A number in the range from -1 to +1 which indicates the extent to which an independent variable (p. 69) accounts for the variability in the dependent variable. The further the coefficient is from zero the greater the extent to which the independent variable accounts for the variability in the depenpent variable. If the coefficient is positive it indicates that as the independent variable increases the dependent variable increases. If it is negative
it indicates that as the independent variable increases the dependent variable decreases and vice versa. See also: Coefficient of determination.

## Coefficient of Determination

A number in the range from 0 to +1 which is a measure of the proportion of the total variation in the dependent variable (p. 69) which can be explained by variation in an independent variable. It is computed by squaring the coefficient of correlation. It is usually preferred to the coefficient of correlation as a measure because it states more clearly the proportion of the variation in the dependent variable which is associated with the independent variable. For example, if 70 percent of the variation in the dependent variable can be explained by the independent variable, the coefficient of determination would be . 7 but the coefficient of correlation would be the square root of .7 (.84). The coefficient of determination is referred to as the index of determination in the case of curvilinear relationships. See also: Coefficient of correlation.

## Coefficient of Variation

A measure of dispersion in the form of a number which is not influenced by the scale of measurement; computed as the ratio of the standard deviation (p. 64) to the arithmetic mean (p. 8). Because of differences in scale of measurement, standard deviations cannot be compared as easily as can the coefficients of variation.

Combinations and Permutations

In mathematics and statistics, a combination is a group of several things or symbols in which the order of arrangement is immaterial. A permutation is an arrangement reflecting a change in order or sequence, especially the making of all possible
changes. Thus, when a problem concerns groups without any reference to order within the group it is a problem in combinations. When the problem requires that arrangements be taken into account it is a problem in permutations. Example: the group of letters ABC make a single combination, whatever their order, but make six permutations, viz. ABC, $\mathrm{ACB}, \mathrm{BCA}, \mathrm{BAC}, \mathrm{CAB}, \mathrm{CBA}$.

## Confidence Coefficient

A measure of the assurance with which an interval estimate (p. 30) may be made. It represents the likelihood that the true population (p. 49) value being estimated lies within the confidence interval (see below) being cited. It is usually expressed as a percentage, and often it is referred to as a confidence level, for example "*** at the $95 \%$ confidence level." There is a tradeoff between the degree of confidence that the true value is within a stated range and the acceptable width of the range. Over specification of the confidence, for example, $99.7 \%$, may result in an excessively wide range, or in other words a very imprecise estimate. A more satisfying combination of precision (p. 49) and confidence, might be found by a modest reduction of confidence, for example, to $90 \%$. The degree of confidence specified will depend upon judgments about the risks associated with an estimate which misses the true value by more than the indicated range of values.

## Confidence Interval

The range of values within which, with a specified level of confidence, the mean ( $p$. 37) value of the population (p. 49) is believed to be contained, based upon the results obtained from a sample (p. 59) of that population. See also: Confidence coefficient.

## Confidence Level

See: Confidence coefficient.

A Federal budget which shows receipts from and payments to the general public, that is, all nonFederal units, including trust funds and some Federal Government sponsored enterprises. A deficit shows that the public is accumulating cash or government securities. Such a budget stresses the cash needs of the Government.

## Consumer Price Index

An index number (p. 29) which is computed monthly by the Bureau of Labor Statistics and which is designed to show the average change in the prices of consumer goods in the United States. The items used in the development of this index are those which experience has shown to be most important in consumer spending. The index measures the average change in retail prices for a "market basket' of goods and services in major urban areas.

## Consumer's Surplus

In economics, the difference between the price that a consumer pays for a good or a service and the amount that he would be willing to pay rather than be deprived of the good or service.

## Contingency Analysis

A procedure employed as a result of uncertainty as to major aspects of the environment assumed in an analysis. The procedure is to vary the assumptions regarding major aspects of the environment and examine the results of the analysis in light of these changed assumptions. An example would be, in an analysis designed to disclose a preferable military strategy from among several alternatives, the use of an assumption that one of our major allies became allied with our potential enemies. See also: Sensitivity analysis.

See: Probability distribution.

## Correlation Analysis

A statistical technique which relates a dependent variable (p. 69) to one or more independent variables in order to determine the extent to which and the direction in which change in the dependent variable is associated with change in the independent variable. When one independent variable is involved, the relationship is called simple correlation. If two or more independent variables are involved, the relationship is called multiple correlation. See also: Regression analysis; Coefficient of correlation.

## Cost-Based Budgets

Budgets in which activity levels are to be estimated in terms of value of resources to be consumed in carrying out the activity, rather than in terms of obligations (p. 42) incurred or payments made. These resource requirements, when distributed to program elements and categories, provide a cost basis for PPB.

Cost-Benefit Analysis
(Benefit-Cost Analysis)
An analytical approach to solving problems of choice. It requires the definition of objectives, identification of alternative ways of achieving each objective, and the identification, for each objective, of that alternative which yields the greatest benefit for a given cost or that alternative which produces the required level of benefits at the lowest cost. This same analytical process has also been referred to as cost effectiveness analysis when the benefits or outputs of the alternatives cannot be quantified in terms of dollars. However there is increasing interest in
combining non-economic benefits with dollar benefits in evaluating particular programs, and methods for doing this are being developed.

Cost-Effectiveness Analysis
See: Cost-benefit analysis.

## Cost Estimating Relationship (CER)

Any numerical relationship which is useful in computing estimated costs of materials or activities. These relationships range from simple averages and percentages to complex equations derived by regression analysis (p. 55) which relate cost (dependent variable) to physical and performance characteristics (independent variable). Estimated costs of an aircraft airframe, for example, may be determined by regression analysis to be a function (p. 26) of airframe weight, delivery rates, and speed. The CER shows how value of these independent variables are converted into costs.

CPM and PERT

CPM (Critical Path Method) and PERT (Program Evaluation and Review Technique) are network analysis (p. 40) models. Each has its own modeling language, but they differ in only one fundamental respect: CPM seeks to determine the expected times of completion of the total project and times of completion of the subprojects of which it is composed. PERT goes further and seeks to estimate variances associated with these expected times of completion.

## Criteria

Measurements which are used to examine the relative degrees of desirability among alternatives (p. 6) or the degree to which a course of action meets an intended objective.

The expression of the relationship between the program structure (p. 53) and the appropriation/budget structure. A crosswalk can be viewed as a table, the stub (rows) of which lists program categories and the columns of which show appropriations and budget activities.

Curvilinear Regression

See: Regression analysis.
Decision Theory
A body of knowledge and related mathematical techniques which have been developed from the fields of statistics, mathematics and logic and which are designed to be of aid in the making of decisions under conditions of uncertainty (p. 69). Decision theory is quite similar to game theory (p. 27) in several respects but a major difference between the two is that in game theory the decision is being made vis-a-vis an opponent, whereas in decision theory the only opponent is nature with its related uncertainty. See also: Hurwicz criterion; Laplace criterion; Maximax criterion; Maximin criterion; and Regret criterion.

Degrees of Freedom
Refers to the size of a sample (p. 59), which is labeled "n," less the number of parameter (p. 45) estimates "used up" in the process of arriving at a given unbiased estimate. For example, to estimate the mean (p. 37) needed to calculate the variance (p. 70) of a population (p. 49) it is necessary to use the mean of the sample, thus using up one degree of freedom. The estimate of the population variance would thus have $n-1$ degrees of freedom. As the number of degrees of freedom is reduced the related variance and thus the standard deviation (p. 64) are increased.

A recently developed technique, the purpose of which is to arrive at a consensus regarding an issue under investigation. It consists of a series of repeated interrogations, usually by means of questionnaires, of individuals whose opinions or judgments are of interest. After the initial interrogation of each individual, each subsequent interrogation is accompanied by information regarding the preceding round of replies. The individual is thus encouraged to reconsider and, if appropriate, change his previous reply in light of the replies of other members of the group.

## Demand

Usually means "demand schedule" which is the relationship between price and quantity demanded. The demand schedule expresses how much of the good or service would be bought at various prices at a particular point in time。

Sometimes changes in the quantity demanded are confused with changes or shifts in the demand schedule. A shift in the demand schedule may mean, for example, that consumers will demand more of the good or service at all possible prices than they would have previously demanded at the same prices. On the other hand, an increase in the quantity demanded would result only by decreasing the price of the good or service。

## Depreciation

Depreciation is a reduction in the value of assets, usually because of wear, aging, obsolescence, and so forth. Depreciation accounting is a system of accounting which aims to distribute in a systematic and rational manner the cost or other recorded value of tangible capital assets, less salvage value, over
the estimated useful life of the assets. Such accounting is a process of cost allocation, not of asset valuation.

## Determinant

A single number which results from solution of a matrix (p. 36) in which the rows and columns represent the coefficients ( p . ll) of a set of simultaneous linear equations, that is, equations which at a given time represent different linear relationships of the same variables (p. 69).

## Difference Equation

An equation which describes the value of a variable (p. 69) in one period in terms of the value of either that variable or another variable in some previous period. If a random variable ( p .54 ) is present, the equation is called a stochastic (p. 66) difference equation.

## Differential Cost

See: Incremental cost.

## Diminishing Marginal Utility

The principle that, as the level of consumption of a good is increased, a point is reached where each additional unit consumed provides less utility than did the preceding unit.
$\frac{\text { Diminishing Returns }}{\text { (Variable Proportions)--Law of }}$

The economic principle that, as there is an increase in the quantity of any variable input which is combined with a fixed quantity of other inputs, the marginal physical product (p. 34) of the variable input must eventually decline. For example, additions of capital to a fixed quantity of labor may result in an
increase in total output，but eventually the addi－ tional output and then the average output associ－ ated with each unit of the variable input（capital） will begin to drop．

## Disbursements

Payments by check or cash for operations and ac－ tivities of the Federal Government，including ad－ vances．

## Discounted Cash Flow

See：Present value。

## Discounting

See：Present value。

## Discrete Distribution

See：Probability distribution。

## Distribution

See especially：Normal distribution．See also： Probability distribution；Frequency distribution；bi－ nomial distribution．

## Dynamic Programming

A mathematical technique which is used in the solution of multistage problems，that is，problems involving many sequential decisions．The objec－ tive is to determine an optimal decision at each stage with＂optimal＂referring to the entire prob－ lem or process rather than merely to the specific stage involved．

## Economic Efficiency

That mix of alternative factors of production（re－ sources，activities，programs，and so on）which
results in maximum outputs, benefits, or utility for a given cost; alternatively, it represents the minimum cost at which a specified level of output can be maintained. Often, because of numerous constraints on costs, programs and activities, the term is used to refer to an alternative which is more efficient than another alternative, thus program A may be referred to as being more economically efficient than alternative program B。

## Economic Good

A physical object which is both useful, in the sense that it satisfies a want or need, and relatively scarce. Both qualities are necessary. Air, while useful, is not scarce, and is not an economic good. Poison ivy, while relatively scarce, is not useful, and is not an economic good. See also: Free good.

Economic Order Quantity (EOQ)
That size order which minimizes the total cost of ordering and carrying inventory. The formula used in computing the $E O Q$ is also known as the optimum lot size formula, or Camp formula. The items considered in the computation include the unit price of the item, the costs of processing each replacement order, and costs associated with carrying inventory such as interest, storage cost and obsolescence.

## Economies of Scale

Factors that reduce average production costs as the size of a plant increases. Economies of scale may be classified either as (1) internal, resulting from the increased size of an individual firm, or (2) external, resulting from the increased size of an industry as a whole.

## Effectiveness

The performance or output received from an approach or a program. Ideally it is a quantitative measure which can be used to evaluate the performance level achieved in relation to criteria (p. 16) pertaining to end objectives. An example of such a measure would be the increase in annual earnings of a group of participants in a Federal retraining program. This example assumes that an objective of the retraining program is to increase the level of income of program participants. Under this assumption, a measure of output, such as the number of people who completed the program, while informative, would not be a valid measure of effectiveness since the objective is to increase income, not merely to retrain people.

## Elasticity

In economics, a measure of the responsiveness of the quantity demanded or supplied to changes in price. Elasticity measures the degree to which price is effective in calling forth or holding back quantity.

Endogenous Variable
A variable (p. 69) the magnitude of which is dependent on and determined by, the model being studied. See also: Exogenous variable.

Engineering Method of Cost Estimating
A traditional means of cost estimating which depends on a well-defined description of a proposed system, availability of detailed bills of material, detailed operations, and specialized judgment. The method produces good results for systems involving standard components and no high-risk developments.

A variable (p. 69) which is wholly independent of the model being studied, that is, a variable determined by outside influences. It is sometimes referred to as "Random Disturbance." See also: Endogenous variable.

## Expected Value

A key concept in probability theory, statistics, and decision theory (p. 17). The expected value of an experiment or a strategy is obtained by multiplying the value associated with each possible outcome of the experiment or strategy by the probability (p. 50) of achieving that outcome and then summing the products of these multiplications. Faced with several possible strategies a decisionmaker who is neither very conservative nor much of a gambler might be expected to choose the strategy having the highest expected value. The concept is also referred to by the term "mathematical expectation."

An example of the calculation of expected value would be an oil company which is considering drilling at a specific location. Assuming that drilling costs are $\$ 50,000$, the probability of finding no oil is $50 \%$, the probability of finding enough oil to assure a net payoff of $\$ 500,000$ is $10 \%$, and the probability of obtaining a net payoff of $\$ 200,000$ is $40 \%$, the expected value calculation would be made as follows:

Possible Payoff

| $\$-50,000$ | .5 | $\$-25,000$ |
| ---: | ---: | ---: |
| 500,000 | .1 | 50,000 |
| 200,000 | .4 | 80,000 |

Expected value of the decision to drill

See also: Laplace criterion.

See: Accrued expenditures.

## Exponential Function

In general, an exponential function (p. 26) is a constant $\mathrm{b}_{\mathrm{x}} \mathrm{e}_{\mathrm{x}}$ raised $\mathrm{to}_{\mathrm{x} 2}$ a variable power, for example $e^{x}, 2^{x}$, and $3^{x 2}$. In other words, it is a function in which the independent variable (p. 69) appears in the exponent of a constant.

## Externality

An economy or diseconomy, that is, a benefit or an undesirable effect, which accrues to an entity as a result of an action which is initiated by another entity and over which the recipient entity has no control.

## F Distribution

A probability distribution (p. 51) which represents the frequency distribution ( $p .26$ ) of the ratios of all possible pairs of estimates of the variance ( $p$. 70) of a normally distributed population (p. 49). It is used to estimate, on the basis of the ratio of their variances, whether two samples have been drawn from a single population or from two separate populations, or, in other words, if the classification such as age or race makes a significant difference in the item being studied such as test scores or job preferences.

## Factor Analysis

A technique for analyzing the interrelationships among various indices. It proceeds on the premise that underlying a larger number of operational indices may be a smaller number of conceptual variables (p. 69) or factors. The object is to replace the large number of indices which may have little theoretical
meaning by the smaller number of variables which may make sense theoretically. It is especially useful where numerous indices may be computed but where underlying theory is unclear. An example would be where a variety of population survey data comprised of indices on such things as occupation, education, rent, fertility, labor force status, housing, and race might be reduced to as few as three underlying factors: economic, family, and ethnic status.

## Fiscal Drag

A phenomenon which involves a retardation in the growth of private purchasing power as a result of Government expenditures not keeping pace with increases in tax revenues. The increases in tax revenues are those which come about as a result of economic growth.

## Fiscal Policy

Federal Government economic stabilization policies designed to foster economic goals such as high employment, stable growth and prices, and balance of payments equilibrium, through changes in taxes and levels of Government spending as distinctfrom monetary policy (p. 39).

## Fixed Cost

A cost which does not change in total within each specified time period or within a range of operations but which increases on a per-unit basis when the volume of production or operations decreases and decreases on a per-unit basis when the volume of production or operations increases. For example, depreciation expense of a building is ofter considered to be the same during each accounting period and thus would be a fixed "cost." While a theoretical distinction is often made between "cost" and "expense," the latter referring to the expired
(consumed) costs of resources acquired, the distinction is not always made by nonaccountants and nonanalysts. See also: Variable cost.

## Free Good

A good (or service) that is so abundant, in relation to the demand for it, that it can be obtained without paying money, without exchanging another good, or without self-exertion. See also: Economic good.

Frequency Distribution
A listing, often appearing in the form of a curve on a graph, of the frequency with which possible values of a variable have occurred. For example, it might show that in a group of 100 persons 50 were within the 10 to 25 year-old category, 30 were within the 26 to 50 year-old category, and 20 were within the 51 to 80 year-old category. Viewed in another way this frequency distribution would show that the variable "age" assumed a value from 10 to 25 years, 50 times, a value from 26 to 50,30 times, and so on. See also: Probability distribution.

## Full (High) Employment Surplus

The budget surplus or deficit that would exist under current tax rates and Government expenditure amounts and policies if the country were operating at full employment (usually meaning 96 percent of the labor force employed).

## Function

A basic mathematical concept. A variable (p. 69) $y$ is said to be a function of another variable $x$ if a rule or relation exists whereby when a value is assigned to $x$, one or more values of $y$ are determined. In this case $x$ would be the independent
variable and $y$ would be the dependent variable. A variable may be a function of more than one variable. An example of this situation would be in the formula for distance traveled, in which distance is equal to the product of velocity and time. In this case distance is a function of both velocity and time.

## Game Theory

A branch of mathematical analysis developed by von Neumann and Morgenstern to study tactical and decisionmaking problems in conflict situations. It is a mathematical process for selecting an optimum strategy in the face of an opponent who has a strategy of his own. Optimality may be defined by any of several criteria.

## Gaming

A type of simulation (p. 61) in which persons, sometimes with the aid of calculating machines, act out roles as decisionmakers in the system being analyzed. In some applications such as that of wargaming the "players" may be grouped into opposing teams. In other applications experts in different fields may "play" the game as independent agents having responsibility for specific segments of the system (for example a regional economy) being studied.

Gaussian Distribution

See: Normal distribution.

## Geometric Mean

An average calculated by multiplying all of the quantities being averaged and then finding whatever root of this product is indicated by the number of items which were multiplied together. For example, if two items were involved the desired root
would be the square root. If three items were involved it would be the cube root and so on. The geometric mean is suitable for averaging ratios and therefore is frequently used in the computation of index numbers (p. 29). See also: Mean.

## Gross National Product (GNP)

The total market value of all final goods and services produced in the Nation in one year. The GNP was about $\$ 860$ billion in 1968 and is estimated to be about \$920 billion in 1969.

## Heuristic

Solution of a problem by a trial and error approach frequently involving the act of learning, and often leading to further discovery or conclusions without providing proof of the correctness of the outcome.

## Hurwicz Criterion

In decision theory (p. 17) this criterion seeks to base decisions under uncertainty (p. 69) somewhere between the conservative maximin (p. 36) (or mini$\max$ ) and the optimistic maximax (p.36) principles. The criterion employs a device called the coefficient of optimism (alpha) which measures the expectations of the decisionmaker on a scale from loptimistic) to 0 (pessimistic). Maximum payoffs are multiplied by the coefficient and minimum payoffs by 1 minus the coefficient. The criterion is the sum of these products and the strategy with the maximum sum is selected.

## Incremental Cost

The cost associated with a change in the level of output. For example, if presently the total cost of production is $\$ 100,000$ and under a planned increase in volume the total cost would be $\$ 125,000$, the
incremental cost would be $\$ 25,000$. The term differential cost is also used to refer to this type of cost.

Index Number

A device used to measure the difference in magnitude of a variable (p. 69) such as price and cost from that which existed during a designated base period. See also: Consumer price index and Wholesale price index.

Index of Determination

See: Coefficient of determination.

Indifference Curve

A graphical representation of alternative combinations of two variables (p. 69) that are related to a constant value of a third variable. The "curve of equal effectiveness" on the cover of this glossary is an indifference curve. In that case each point along the curve represents a combination of programs G and B which would produce a specified level of effectiveness, that is, the level of effectiveness is the same no matter which point along the curve (combination of the two programs) is selected. The slope (p. 62) of an indifference curve is known variously as "the marginal rate of substitution," "the substitution ratio," and "the relative marginal utility ratio." See also: Iso-cost curve; Iso-quant curve; Iso-utility curve.

Input-Output (Interindustry,
or Leontief) Analysis
A systematic technique for quantitatively analyzing the interdependence of producing and consuming units in an economy. It studies the interrelations among producers as buyers of each other's outputs, as users of scarce resources and as sellers to
final consumers. The technique has been useful for the study of the way in which the component parts of an economy fit together and influence each other, for short-run forecasting, and for policy guidance.

## Integer Programming

A technique of linear programming (p. 32) in which the decision variables and thus possible solutions can assume only integer values. This is frequently required in practice since half a plane cannot fly and half an auditor cannot travel. However, the integer values do not result from simple rounding of the normal linear programming iterative processes.

## Internal Rate of Return

See: Present value.

## Interval Estimate

An estimate which states that the characteristic of interest has a value that is located somewhere within a range or interval of values. See also: Point estimate; Confidence coefficient.

## Iso-cost Curve

An indifference curve (p. 29) which indicates graphically the various combinations or mixes of two outputs that can be obtained for a specified cost.

Iso-quant Curve

An indifference curve (p. 29) showing the different combinations of two factors of production that will yield the same physical product. An example is the "curve of equal effectiveness" on the cover of this glossary. In that case the factors of production are programs G and B and the physical product is "effectiveness." See also: Iso-utility curve.

An indifference curve (p. 29) which indicates graphically the various combinations of two commodities which will yield the same amount of utility to a consumer. A series of iso-utility curves involving the same two commodities drawn to represent the utility valuations of one consumer is referred to as an indifference map.

Iterative Process
A process for calculating a desired result by means of a repeating cycle of operations, which comes closer to the desired result with each repetition.

## Laplace Criterion

In decision theory (p. 17) this criterion bases decisions under uncertainty upon the assumption that, because the probabilities of future states of nature are unknown, they should be considered to be equal. If a decision is based on the Laplace criterion, equal probabilities are assigned to each possible state of nature, and the alternative which maximizes expected value (p. 23) is selected. The Laplace criterion is sometimes called the "principle of insufficient reason."

## Learning (or Progress) Curve

A curve which describes the set of points conforming to the observed phenomenon that cost reductions yield a constant percentage decrease for each doubling of the cumulative quantity produced. This means that the cost of manufacturing unit 2 will be a certain percentage less than the cost of manufacturing unit 1 ; the cost of unit 4 will be the same percentage less than unit 2 , and so on.

As applied to functions (p. 26) in general it is a basic tool of mathematics that deals with the value approached by a function as its independent variable (p. 69) approaches some fixed value.

## Linear Programming

A deterministic model (p. 38) which assumes linear behavioral relationships and in which an optimal solution is sought (maximizing or minimizing) subject to one or more limiting constraints. Linear programming is used to determine the best or optimum use of resources to achieve a desired result when the limitations on available resources can be expressed by simultaneous linear equations. Every solution has a primal and a dual aspect, that is, a solution maximizing something (primal) and also one minimizing something (dual). That which is to be maximized or minimized is called the objective function. In business the objective function may be profit; in Government agencies the objective may be minimization of costs or maximization of program output within a specific cost constraint.

A systematic, iterative, but still trial and error method of solving linear programming problems where (1) the number of variables prohibits graphical methods or simple algebraic solutions; and (2) the "stepping stone" trial and error method is inappropriate or infeasible is called the simplex method. It starts with one possible solution and makes a systematic search for a superior one; then the procedure is repeated.

## Lorenz Curve

A graphical depiction of the distribution of something, usually income or wealth, among a group of
people. It is derived by plotting the cumulative proportion of people, from the poorest up, against the cumulative share of total income which they receive. If everyone received the same income, the Lorenz curve would be a 45 degree straight line. See the illustration below.


## Macroeconomics

The division of economics which deals with aggregates such as total income, total output, total employment, the general price level, and the general level of wages, in the economy being studied, which is usually the national economy. Macroeconomic analysis may also be referred to as aggregate economic analysis or income and employment theory. See also: Microeconomics.

The process of identifying the benefits or costs of alternative behaviors as unitary changes in the alternative variables occur and equalizing the benefitcost ratio (p. 9) to form a point of indifference (trade-off) in benefit per additional unit of resources input for decisionmaking purposes. See also: Indifference curve.

## Marginal Conditions

Conditions which must be fulfilled if an operation is to attain an optimum, for example that marginal revenue equal marginal cost is a "first-order condition," that the excess of marginal revenue over marginal cost must have been declining just prior to this point of equality is a "second-order condition."

## Marginal Cost

In a marginal analysis (see above), the change in total cost due to a one unit change in output. It is a special case of the more general term incremental cost (p. 28). Theoretically, a purely competitive firm will maximize profits by increasing output until marginal cost equals price, while an imperfectly competitive firm will equate marginal cost to marginal revenue (p. 35).

Marginal Cost Per Unit of Input or MCI
(sometimes called Marginal Revenue Cost or MRC)
The change in total cost resulting from a one-unit change in a variable input (p. 70).

Marginal Physical Product or MPP
The change in total output resulting from a oneunit change in a variable input (p. 70).

The change in total revenue due to a one-unit change in output. See also: Marginal Cost.

Marginal Revenue Per Unit of Input or MRI (sometimes called Marginal Revenue Product or MRP)

The change in total revenue resulting from a oneunit change in a variable input (p. 70).

## Marginal Utility

The change in total utility due to a one-unit change in the quantity of a good or service consumed, for example, the additional satisfaction that a purchaser derives from buying an additional unit of a commodity or service. Marginal utility is a psychological rather than an objectively measurable concept.

Markov Analysis

A method of analyzing the current movement of some variable (p. 69) in an effort to predict the future movement of that same variable. A firstorder Markov process is based on the assumption that the probability (p. 50) of the next event depends on the most recent event and not at all on any previous event. A second-order Markov process assumes that the next event depends on the past two events, and so on. A simple example of a firstorder Markov process would be a baseball team's performance, if it could be shown that the key to determining the probability of $a$ win is the result of the preceding game, that is, that if the team won its last game the probability of a win today is .6 but if it lost yesterday the probability of a win is . 4 .

See: Expected value.

Matrix

A rectangular array of rows and columns of real numbers. Matrices may be subjected to mathematical operations such as multiplication of one by another, addition of two or more, and others. Matrices may be manipulated in total in a manner similar to the algebraic manipulation of single numbers, but knowledge of special rules, called matrix algebra, is necessary for such manipulation. The development of matrix algebra and of computer solution has made possible the efficient solution of very large systems of simultaneous linear equations. An illustration of a matrix may be found on page 58. See also: Determinant; Vector.

## Maximax Criterion

In game theory (p. 27) or decision theory (p. 17) this criterion states that under uncertainty (p. 69) the decisionmaker should select that strategy which for one of its possible outcomes has the highest value payoff of all the payoffs associated with the outcomes of all possible strategies. For example if strategy A has potential payoffs of 5,10 , or 15 ; strategy $B$ has potential payoffs of $8,9,10$, and strategy $C$ has potential payoffs of 0,5 , and 17 , this criterion would dictate the choice of strategy C, since 17 is the highest of all the possible payoffs or in other words, the maximum of the maxima. See also: Hurwicz criterion; Laplace criterion; Maximin criterion; and Regret criterion.

Maximin Criterion (Wald Criterion)
In game theory (p. 27) or decision theory (p. 17) this criterion states that under uncertainty (p. 69) the decisionmaker should select that strategy whose lowest possible payoff is higher than the lowest possible payoff of any other strategy. For example,
if strategy A has possible payoffs of 5, 10 or 15; strategy $B$ has possible payoffs of 4,10 , and 12 ; and strategy $C$ has possible payoffs of 3,25 , and 30 ; this criterion would dictate the choice of strategy A, since 5 is higher than either of the other two lowest possible payoffs or in other words it is the maximum of the minima. This definition has assumed that the decisionmaker's intention is to maximize. If the problem had been one of minimization (such as of costs) and the payoffs as listed had been in terms of that which was to be minimized this same criterion would have dictated the choice of strategy $B$, since 12 is the lowest of the three high payoffs, or in other words it is the minimum of the maxima. In minimization problems therefore this criterion is called the minimax criterion. Hopefully it can be seen at this point that this criterion dictates highly conservative strategy. See also: Hurwicz criterion; Laplace criterion; Maximax criterion; and Regret criterion.

Mean

One of the kinds of averages. The mean (also called arithmetic mean) is what is usually connoted by the term "average." It is computed by summing the values of the items being observed and dividing this sum by the number of observed items.

Median

One of the kinds of averages. The median is that value above which $50 \%$ of the items in the group being observed fall and below which $50 \%$ of the items fall.

Microeconomics

The division of economics which is concerned with the income of a firm, the output of a firm or single
industry, the price of a single commodity or service, the wage rate of an individual worker or the wage bill of one firm or industry. $\operatorname{PPBS}$ (p. 48) and systems analysis (p. 67) draw heavily upon the analytic tools of microeconomics. See also: Macroeconomics.

Minimax Criterion

See: Maximin criterion.
Mode

One of the kinds of averages. The mode is that value which appears most frequently in the group of data being observed.

Model

A representation of the relationships that define a situation under study. A model may be a set of mathematical equations, a computer program, or any other type of representation, ranging from verbal statements to physical objects. Models permit the relatively simple manipulation of variables (p. 69) to determine how a process, object, or concept would behave in different situations.

## Deterministic Model

A model in which the variables (p. 69) take on only definite values, that is, a model that does not permit any risk as to the magnitude of the variables. For example, a set of simultaneous equations for which there is a unique solution.

## Probabilistic Model

A model in which each variable may take on more than one value. Such models are sometimes called stochastic (p. 66) which means, literally, "making a best guess."

## Monetary Policy

> Federal Government economic stabilization policies, primarily executed by the Federal Reserve System, designed to achieve economic goals such as high employment, stable growth and prices, and balance of payments equilibrium, through influence on the money supply, interest rates, and credit availability, as distinct from fiscal policy (p. 25).

## Monopoly

A market situation characterized by (l) a single seller who sets his own price independently, and (2) a large number of unorganized buyers. See also: Monopsony.

## Monopsony

A market situation characterized by (1) a single buyer who sets his own purchase price independently, and (2) a large number of unorganized sellers. See also: Monopoly.

## Monte Carlo Methods

A catch-all label referring to methods of simulated sampling. When taking a physical sample is either impossible or too expensive, simulated sampling may be employed by replacing the actual universe (p. 69) of items with a universe described by some assumed probability distribution (p. 51) and then sampling from this theoretical population (p. 49) by means of a random number table.

## National Income Analysis

An important tool of macroeconomics (p. 33). It is the measurement of the aggregate flows of the economy (for example income, consumption, investment) and study of the relationships among these flows.

A technique used in the planning, scheduling, and solving of problems related to large-scale projects which involve a great number of interrelated decision points or events. The project is displayed as a network which connects these points or events in such a way as to show the various alternative "paths" which may be taken from each point. When used in scheduling, each event is dependent upon certain necessary events having precededit. These dependencies are portrayed by the connecting aspects of the network. See also: CPM and PERT.

## New Obligational Authority (NOA)

The total of all budget authorizations, provided by current or prior actions of the Congress, of whatever type, of a given year which provide new authority to incur obligations. NOA has consisted of appropriation, contract authority, or authority to spend debt receipts. In the future the term "appropriations" will be applied to this concept of NOA.

## Nomograph

A graph which enables one, with the use of a straightedge, to determine the value of a dependent variable (p. 69) by locating and joining the values of two independent variables. In the illustration below, for example, if the value of $x$ (speed) is 47 and the value of $y$ (weight) is 65, by use of a straightedge it can be determined that the value of $z$ (labor costs) is 185. Construction of a nomograph, such as the one in this illustration, may require use of regression analysis (p. 55) to determine the relationships between the variables, that is, the extent to which a change in the value of one variable, the independent variable, tends to be accompanied by a change in the value of another variable--the dependent variable.


## Normal Distribution

The most important probability distribution (p. 5l) in statistics. It is of the continuous type and is usually represented by a symmetrical bell-shaped curve. Its great value lies in the fact that it describes the distribution of random sampling results for most populations (p. 49). Therefore it can be used to compute the probability (p. 50) that the value of the statistic (p. 65) for example, the mean (p. 37) of the sample or the standard deviation (p. 64) of the sample, is within a calculable interval around the population value which is being estimated. This probability is referred to as the confidence coefficient (p. 13) and the calculated width of the interval at any specific probability or confidence level is the measure of the degree of precision (p. 49) of the interval estimate (p. 30) being made as to the population value of interest.

See: Welfare economics.

## Null Hypothesis


#### Abstract

A hypothesis which is formulated with the intent and hope that the data which is gathered will be sufficient to enable the scientist to reject the hypothesis. It is usually easier to obtain sufficient evidence to convince someone that he should reject a hypothesis than it is to obtain enough evidence to convince him to accept a hypothesis. For this reason the usual procedure in scientific investigation is to formulate a null hypothesis which states the opposite of what the investigator believes or hopes to be the case. The investigator then attempts to gather sufficient evidence to be able to reject the null hypothesis.


## Objective Function

See: Linear programming.

## Objectives

Goals or results that the decisionmaker wants, or should want, to attain. Hence, the end product or output of a program.

## Obligations

Obligations in Federal accounting represent commitments to acquire materials or services or to make payments under certain conditions (such as loans, grants, subsidies, and contributions). The Congress has specifically prescribed the kinds of transactions that may be recorded and reported as obligations of the Government of the United States. Total obligations incurred would thus be the amounts of orders placed, contracts awarded, services received, and similar transactions requiring
the later disbursement of money. In contrast, under the accrual basis of accounting, expenditures represent performance or incurrence, within a given period of time. See also: Accrual accounting.

## Ogive

A curve which is derived from a frequency distribution (p. 26) and which portrays the percentage of the data points which are above or below any value. An ogive which shows the percentage of data points below each value rises from left to right. An ogive showing the percentage of data points above each value descends from left to right. The value at which a "less than" ogive and a "more than" ogive, derived from the same frequency distribution, intersect is the median (p. 37) of that distribution. The illustration below was derived from a frequency distribution of a hypothetical group of people and shows the percentage of the total population above and below any age. In the illustration the median age is 30 .


Conceptually, a mix of activities and resources under common management which represents the most detailed organizational or budgetary level whose identification is required in the information system. The operating program may be identical with a program element (p. 52) if its purpose can be identified by only one program category (p. 52). Alternatively, the operating program may serve more than one such purpose, in which case each part of the operating program identified by a discrete program category is a program element.

Operations Research (OR)
The use of analytic methods adopted from mathematics and other disciplines for solving operational problems. Among the common techniques used in operations research are: linear programming ( $p$. 32), probability theory, decision theory (p. 17), Monte Carlo methods (p. 39) and queuing techniques (p. 53).

## Opportunity Cost

The measurable advantage foregone as a result of the rejection of the next best alternative use of resources. For example, the opportunity costs of assigning auditors to undertake a particular examination are the benefits that would have been achieved by assigning the auditors to the next best alternative audit. The opportunity cost related to a decision to have the Government invest funds in an activity would be the benefits which would be derived on those funds if they were allowed to remain in the private sector of the economy.

## Optimum, Solving for the

A mathematical procedure used to determine the best mix of inputs to meet an objective. An optimum
may be derived mathematically by differentiating a function (p. 26) with respect to each variable (p. 69) setting the resulting equations equal to zero, and solving them.

## Outlays

Outlays cover both the expenditures account and the loan account of the Federal Budget. That is, outlays are disbursements (p. 20) from the expenditures account and net lending (loans less repayments) from the loan account. It is contemplated that in the 1972 budget outlays will include accrued expenditures (p.4) in lieu of disbursements.

## Out-of-Pocket Costs

Costs associated with the decision under consideration which require outlays. They may be contrasted with those costs such as depreciation (p. 18) which are in reality allocations of previously incurred costs. See also: Fixed cost; Variable cost; Opportunity cost; Marginal cost.

## Parameter

A characteristic which is considered to be essential in accurately describing a problem, population (p. 49), or system and the value of which is therefore held constant during a calculation related to that problem, population, or system. In this sense, it may be contrasted with a variable (p. 69) which can assume any of a large number of values. An example of the use of parameters would be in statistical inference (p. 65) where in estimate calculations such population characteristics as the mean (p. 37) and the standard deviation (p. 64) may be assigned specific values based on the results found in a sample (p. 59). In this case the mean and the standard deviation would be parameters in the calculations.

Pareto Optimum
In welfare economics, (p. 71) a concept that sets the conditions that maximize the economic wealth
of a given society. The pareto optimum is said to have been achieved when it is impossible to make one person better off without making another (or others) worse off. Other economists, for example Kaldor, Hicks and Scitovsky, define the optimum somewhat differently.

## Partial Correlation Coefficient

A number in the range from -1 to +1 which indicates the extent to which an independent variable (p. 69) accounts for the variability in the dependent variable after adjusting for the effect of another independent variable on this relationship. See also: Coefficient of correlation.

## Payback Period

The length of time required for the stream of cash proceeds produced by an investment to equal the original cash outlay required by the investment. One of several project evaluation methods. Generally considered by analysts to be inferior to the present value method (p. 49) because it ignores project benefits and costs once the cash outlay for the investment has been recovered. Also called payoff period and cash recovery period.

## Pearsonian Skewness Coefficient

A number which indicates the relative degree of skewness (p. 61) in a distribution (p. 20). It is obtained by subtracting the mode (p. 38) from the median ( $p$. 37) and dividing the difference by the standard deviation (p. 64) of the distribution.

## Pecuniary Spillover

A spillover (p. 64) which is monetary rather than physical in nature and which causes a change in the monetary valuation of a physical input or output, but does not change the relationship between physical inputs and physical outputs. For example, an acceleration of a man-to-Mars program timetable might cause a short-run shortage of professionals and
technicians thus increasing the costs of similar services to other industries but not necessarily changing the physical productivity of these inputs to the other industries. See also: Spillover and Technological spillover.

## Performance Budget

A budget based upon functions, activities, and projects, whose principal analytical orientation is the measurement of efficiency of operating units. For example, such a budget in an agency might require computation of the cost per unit of mail processed for one branch of the agency and the cost per loan application processed in another branch.

Permutations

See: Combinations and permutations.

## PERT

See: CPM and PERT.

## Planning

Planning in the long-range sense is the selection or identification of the overall, long-range objectives of the organization and the analysis of various possible courses of action in terms of their relative costs and accomplishments or benefits in order to decide on which courses of action (such as programs) to follow in order to achieve those objectives. The analyses required are variously referred to as cost-effectiveness, cost utility, or cost-benefit (benefit-cost) studies. Essentially, this type of planning involves deciding on what the organization is in business to do and generally how it is to be done. This is also called strategic planning.

Short-range planning encompasses all other management planning responsibilities and focuses on studies and analyses made for the purpose of identifying the specific feasible and economical means for carrying out the chosen courses of action.

Planning-Programming-Budgeting System (PPBS)
A management system, introduced in the Executive branch, in 1965 by the President comprised of three elements:

Planning the study of objectives and of alternative ways to achieve objective, of future environments, and of contingencies and how to respond to them.

Programming A method or system of describing activities according to objectives or outputs and of relating these objectives to the costs or inputs needed to produce the outputs or effectiveness desired.

Budgeting The activity through which funds are requested of the President and Congress, appropriated, apportioned, and accounted for.

## Point Estimate

An estimate which states that the characteristic of interest has a single, specific value. It is the single best estimate of a population (p. 49) value made from a sample (p. 59). For example, if a sample discloses that the average dollar error per voucher is $\$ 10$ and there are 47,189 vouchers in the population, the point estimate of the total dollar error is $\$ 471,890$. This may be contrasted with an interval estimate (p. 30) which states that the population value of interest falls somewhere within a range or interval.

## Poisson Distribution

A probability distribution (p. 51) of the discrete type which is used for predictions related to the
occurrence of rare events. It is useful in predicting the number of times that a specific event will occur in a specified time period, for example, the number of cars which will pass an intersection in an hour or the number of telephone calls which will reach a specific switchboard in five minutes. In statistical quality control it is used for predictions concerning the number of defects which will be found per unit. This use may be contrasted with that of the binomial distribution ( p .10 ) which may be used to predict the number of defective units which will be found among 1,000 units.

## Population

In statistics, the total collection from which a sample (p. 59) is to be drawn. Sometimes referred to as the universe.

## Precision

In statistical inference (p. 65), the measure of precision is the size of the interval within which the value being estimated is predicted to be found with a specified degree of assurance, based upon the results obtained from a sample (p. 59). There is a tradeoff between the degree of precision of an estimate and the degree of assurance with which it may be made. If a less precise estimate, that is, one with a wider interval, is tolerable, the degree of assurance or confidence level (p. 13) can be increased. See also: Sampling error.

Present Value (Net Present Value or
Discounted Present Value)
The maximum amount that an investor or agency could pay for or invest in a project without being financially worse off. The present value method of project evaluation is based upon the principle that a dollar received today is worth more than a dollar
received a year from now. It requires the analyst to use an interest rate to discount future benefits and costs to the present. The present value (p.v.) of $x$ dollars which will be received at the end of $n$ years from now, may be computed by use of the formula P.V. $=X\left[\overline{(1-i)}^{m}\right]$ where $i$ is the applicable interest or discount rate. A cost of capital concept is used by commercial enterprises to select the appropriate discount rate whereas the interest rate on Government borrowing and the social opportunity cost discount rate (p. 63) are often advocated for use by agencies of the Federal Government. The present value of $\$ 100$ payable in two years can be defined as that amount of money necessary to invest today at compound interest in order to have $\$ 100$ in two years. Thus present value depends on the rate of interest, the frequency of compounding, and the time horizon selected. A somewhat different approach to the same kind of project evaluation problem is to determine what interest rate will result in the net present value of the project being equal to zero, that is that the present value of project costs is exactly equal to the present value of project benefits. The rate which accomplishes this is called the internal rate of return.

The concept of present value can perhaps be more readily perceived in the following lists of present values of $\$ 1$, under two discount rates, with compounding occurring annually.

|  | Discount Rate |  |
| :---: | :---: | :---: |
| Received at End of | $\underline{5} \%$ | $\underline{10 \%}$ |
| 1 year |  | .95238 |
| 2 years | .90909 |  |
| 3 years | .90703 | .82645 |
| 4 years | .86384 | .75131 |
|  |  | .82270 |
|  |  | .68301 |

Probability

The ratio of the number of outcomes that would produce a specific event to the total number of
possible outcomes or, in other words, the likelihood of a specific event occurring. It is expressed as a number in the range from zero to one or as a percentage.

## Conditional Probability

The likelihood that an event or outcome will occur, given that another event has occurred or a specific condition exists. For example, the probability of getting two heads from two tosses of a fair coin is $1 / 4$, which is arrived at by multiplying $1 / 2$ (the probability of a head on the first toss) by $1 / 2$ (the probability of a head on the second toss). However, the conditional probability of getting two heads given that the first toss was a head is $1 / 2$, that is, at this point since we know that a head resulted from the first toss the only probability of relevance is that of getting one head on one toss which, since the coin is fair, is equal to $1 / 2$.

Subjective Probability
A probability for which historical evidence is not available for decisionmaking. The decisionmaker must therefore rely on his own estimation of the likelihood of various possible outcomes.

## Probability Distribution

A model (p. 38) for the prediction of the relative frequencies with which possible values of a variable (p. 69) will occur. Often it is developed on the basis of and presented in a manner similar to a frequency distribution (p. 26). There are two types: (1) discrete, in which the variable can assume only specific values (usually integer values $0,1,2$. . .) for example, the number shown on a die, or number of baseballs lost in a game, and (2) continuous, in which the variable can assume any value (or any value within a specified interval) for example, the amount of rainfall per month in Boston.

A major agency endeavor, mission oriented, which fulfills statutory or executive requirements, and which is defined in terms of the principal actions required to achieve a significant end objective.

Program and Financial Plan (PFP)

A multi-year budget forecast based on a program structure (p. 53) which projects the future (usually five years) output and cost implications of current decisions and shows comparative data for the fiscal year just past, the current year, and the budget year.

## Program Category

A classification within a program structure (p. 53) which groups programs which have the same or similar objectives. See also: Operating program.

## Program Element

A subdivision of a program category (see above) which comprises the specific products that contribute to an agency's objective (s). If an agency's operating program (p. 44) is distributed over several program categories each part of the operating program identified by a discrete program category is a program element.

Program Memoranda (PM)

Documents which present statements of major program issues, that is issues which require decisions in the current budget cycle, with major implications in terms of either present or future costs, the direction of a program or group of programs, or a policy choice. A program memorandum should include a comparison of the cost and effectiveness of alternatives for resolving the issues being discussed, the
agency head's recommendations on programs to be carried out, and the reasons for those decisions.

## Programming

Programming is the process of deciding on specific courses of action to be followed in carrying out planning decisions on objectives. It also involves decisions in terms of total costs to be incurred over a period of years as to personnel, material, and financial resources to be applied in carrying out programs (p. 52).

## Program Structure

A set of program categories (p. 52), program subcategories (see below) and program elements (p. 52). Usually the term has reference to a single agency.

## Program Subcategory

A subdivision of a program category (p. 52). It combines agency programs or activities on the basis of narrow objectives within the broader objectives of the program category.

## Queuing Techniques

Techniques used when a problem involves providing a supply of goods and services in order to satisfy randomly arriving demands for these goods and services. More specifically, the techniques associated with operations research (p. 44) which determine the amount of delay that will occur when operations (such as supplying goods or services) have to be provided in sequences for objects (such as customers) arriving randomly. Queuing theory may be applied to any operation in which objects arrive at a service facility of limited capacity.

In most cases problems for which these techniques are used involve arrivals at irregular intervals.

If the distribution (p. 20) of the time between arrivals and that of the lengths of time required to provide the service fit recognized mathematical distributions, and the problem for which a solution is sought is not overly complex, optimal solutions may be found by use of queuing techniques. When any of these conditions is absent, however, by the use of Monte Carlo methods (p. 39) simulated data on the time between arrivals and on service times can be generated and the effect of changes in the situation, such as adding to or reducing the capacity of the service facility, can be examined.

Random Disturbance

See: Exogenous variable.

## Random Variable

A quantity, the value of which is determined by the results of an experiment and the possible values of which can be described by a probability distribution (p. 51). For example, random variable H might be assigned the value of the number of heads resulting from 100 tosses of a coin. See also: Stochastic.

## Recursion

The process of repeating an operation or group of operations, usually with the result of each repetition being in some way dependent upon the result of the preceding repetition.

Analysis undertaken to determine the extent to which a change in the value of one variable ( $p .69$ ), the independent variable, tends to be accompanied by a change in the value of another variable (the dependent variable). Where only one independent variable is involved in the analysis the technique is known as simple regression analysis; where two or more independent variables are involved the technique is called multiple regression analysis. If the relationship between two variables can be depicted graphically by a straight line it can be defined mathematically by an equation of the form:

$$
y=a+b x
$$

where $y$ is the dependent variable and $x$ is the independent variable. Multiple regression analysis can similarly be defined by an equation of the form:

$$
y=a+b x_{1}+c x_{2}+d x_{3}+\cdot \cdot \cdot \mathrm{zx}_{n}
$$

but in this case graphical representation is impossible.

If the change in the dependent variable associated with a change in the independent variable does not occur at a constant rate, the regression line takes the form of a curved line and the analysis is referred to as curvilinear regression analysis. Graphical illustrations of simple linear regression and curvilinear regression lines and plotted data points are shown below to illustrate the relationship between the heights of two groups of girls (the dependent variable) and their ages (the independent variable).

Regression lines are drawn or defined in such a way that the sum of the squared deviations (the squares of the vertical distance of each point from the line) is smaller than would be the sum of the squared deviations from any other line which could be drawn. The relationships identified by means of regression analysis are associative only; causative inferences must be added subjectively by the analyst or obtained by other means. See also: Correlation analysis.

## LINEAR REGRESSION




Age in Years

In decision theory (p. 17) this criterion bases decisions under uncertainty (p. 69) upon a computation of the difference between (1) the payoffs realized when various states of nature occur and (2) the maximum payoff possible. The decisionmaker acts to minimize this difference, called regret.

Risk
"Measurable uncertainty" per the economist Frank Knight. In decision theory, (p. 17) the distinction is made that risk is measurable while uncertainty (p. 69) is not. In situations of risk, the probabilities associated with potential outcomes are known. The term may be associated with situations of repeated events, each individually unpredictable but with the average outcome highly predictable. In situations of uncertainty, the probabilities are not known.

## Saddle Point

An element of a matrix (p. 36) that is both the lowest element in its row and the highest element in its column. In game theory, ( p .27 ) the saddle point is called the minimax solution. However, a saddle point is not a necessary matrix requirement, and therefore, does not always exist. In the illustration below the block in which 32 appears is the saddle point.

SADDLE POINT

| 20 | 8 | 10 |
| :---: | :---: | :---: |
| 8 | 17 | 15 |
| $32^{*}$ | 50 | 40 |
| 25 | 19 | 30 |
| 12 | 14 | 26 |

The units concerning which observations are recorded in an experiment. It is a subset of the population (p. 49). See also: Statistic.

## Sample Space

A listing of all possible outcomes of a specific experiment. The form of this listing may be completely narrative, however it is usually pictorial. For example, the sample space for a coin-toss experiment would frequently be represented as a "tree" having two branches, one labeled "head" and the other labeled "tail."

## Sampling Error

In statistics, it is the difference between the value found in a sample (see above) and the value of the same characteristic which would have been found if the entire population (p. 49) had been tested. See also: Bias.

## Satisficing

A term, advanced by Herbert Simon, which views decisionmaking as a process of reaching satisfactory positions rather than optional positions, where the standard of satisfactory is given by complex psychological and sociological considerations.

Scalar

A quantity having magnitude but no direction as contrasted with a vector (p. 70) which has both. It is simply a constant or a number. An example would be body temperature.

Scenario

A narrative description of the environment, scope, and timing of the problem area under analysis.

Benefits from a project that accrue indirectly to an external entity. An example of derived secondary benefits is the increased net income of farmers and others from processing, transporting, and selling products in the area of a Bureau of Reclamation project. An example of an induced secondary benefit resulting from the project would be the net income of a new plant that located in the project area solely because of the project.

Sensitivity Analysis
A procedure employed as a result of uncertainty (p. 69) as to the actual value of a parameter (p. 45) or parameters included in an analysis. The procedure is to vary the value of the parameter or parameters in question and examine the extent to which these changes affect the results of the analysis. For example, if an analysis indicates that program A is preferable to program B, sensitivity analysis might be performed by increasing a factor such as size of the group to which the programs are directed and then examining the results of the analysis under this change. See also: Contingency analysis.

## Shadow Price

An imputed value; an exchange rate other than a market price. In economics (especially in appraisal of public investment projects), the estimates of the intrinsic value of the scarce factors of production available. Shadow prices may be used when market prices (particularly those of capital and labor) diverge from the values that would prevail if (1) the investment under consideration were actually carried out, and (2) no fundamental disequilibria existed in the market.

In linear programming (p. 32), the shadow price is the amount of change in objective achievement per marginal change in some constraint.

See: Linear programming.

## Simulation

An abstraction or simplification of a real world situation. In its broadest sense any model (p. 38) is a simulation, since it is designed to represent the most important features of some existential condition(s). Generally, however, the term simulation is used to refer to a model which is being used to determine results under each of many specific sets of circumstances rather than one which is being used to determine an optimal solution to a problem. Simulations may take the form of either deterministic models or probabilistic models.

Man-machine simulation is simulation in which both calculating machines and human decisionmakers interact in simulating a process or system. Most of these simulations can be legitimately categorized under the heading of "gaming" (p. 27).

Reference to those simulations that are carried out solely by machines is called pure-machine simulation. This is in contrast to man-machine or all-man simulation in which human decisionmakers serve as part of the model.

## Skewness

The degree of deviation from symmetry in a distribution, (p. 20) that is, the extent to which the distribution differs from one in which $50 \%$ of the values are above the mean (p. 37) value and 50\% are below the mean value. See also: Normal distribution.

## Slack Variable

A fictitious variable (p. 69) introduced in the process of solving a linear programming (p. 32) problem
by the simplex method. The reason for its use is to convert an inequality to an equality. For example, if a constraint in the problem were $2 x$ $+5 y<20$, this inequality could be converted into the equality $2 x+5 y+z=20$. In this case $z$ would be the slack variable.

Slope (mathematical)
A measure of the steepness of a line. It is computed by calculating the number of units the line rises per unit move to the right. That is, if moving ten units to the right involves a five-unit rise in the graph as is the case in the illustration below, the slope of the line is $5 / 10$ or $1 / 2$. When a line is used to represent the relationship between two variables (p. 69) the slope may be defined as the algebraic change in the dependent variable (y), which is plotted along the vertical axis, per unit increase in the independent variable ( $x$ ), which is plotted along the horizontal axis, as a point $p$, moves along the line. Thus, if $y$ increases as $x$ increases, the slope is positive and the line rises to the right. If $y$ decreases as $x$ increases, the slope is negative and the line falls to the right, or rises to the left. If $y$ remains constant as $x$ increases, then the slope is zero and the line is parallel to the x axis. Stated in symbols, a number, m, called the slope is defined by the ratio:

$$
\mathrm{m}=\frac{\mathrm{y}_{2}-\mathrm{y}_{1}}{\mathrm{x}_{2}-\mathrm{x}_{1}}
$$

where $y_{1}$ and $y_{2}$ are the values of $y$ at the two points, on the line, between which the slope is being calculated, (if the line is straight any two points may be used) and $x_{1}$ and $x_{2}$ are the values of $x$ at the same two points. In the illustration $x_{1}$ is 10 , $x_{2}$ is $20, y_{1}$ is 5 and $y_{2}$ is 10 .


## Social Opportunity Cost <br> Discount Rate

A discount rate used to measure the value to society of the next best alternative uses to which funds employed in a public investment project might otherwise have been put by taxpayers. In a perfectly competitive economy the cost of such funds would be represented by the market rate of interest. Some economists believe that evaluations of proposals for Federal Government projects require that future costs and benefits be discounted at a discount rate which reflects both the social time preference rate and the productivity of funds in private investment. See also: Social time prefence rate.

Social Time Preference Rate

A discount rate considered by some economists and analysts to be appropriate for use in costbenefit analysis (p. 15) of Federal Government programs. This approach to a discount rate holds
that the discount rate should be a vehicle for expressing objectives which are distinctly public in nature. A form in which this position is frequently set forth is that of the argument that existing market interest rates do not adequately reflect the wishes of future generations and thus are biased in favor of present consumption as opposed to investment for later use. Thus this position argues for a lower discount rate than market interest rates would indicate to be appropriate. The use of a lower rate will tend to make investment for the future appear more desirable from a cost-benefit standpoint. See also: Present value; Social opportunity cost discount rate.

## Spillover

An economy or diseconomy for which no compensation is given (by the beneficiary) or received (by the loser). Spillover is sometimes synonymous with externality (p. 24) and with external economy or external diseconomy. See also: Secondary benefits which is a closely related concept.

## Standard Deviation

A measure of the degree of spread among a set of values; a measure of the tendency of individual values to vary from the mean (p. 37) value. For example, two sets of values $3,4,5,6,7$ and 1,4 , $3,15,2$ have the same mean, 5 , but standard deviations of 1.4 and 5.1 respectively. This difference reflects the fact that the values in the second set are more widely dispersed around their mean than are the values in the second set. The standard deviation is computed by subtracting the mean value from each value, squaring each of these differences, summing these results, dividing this sum by the number of values in order to obtain the arithmetic mean of these squares, and then finding the square root of this mean. The square of the standard deviation is the variance (p. 70).

## Standard Deviation from Regression

See: Standard error of estimate.

## Standard Error of Estimate

A measure of the degree of spread among the values being studied in a regression analysis (p. 55); a measure of the tendency of individual values to vary from the regression line. Conceptually it is quite similar to the standard deviation (p. 64), the difference being that the standard deviation measures the spread around the mean (p. 37) value whereas the standard error of estimate measures the spread around the regression line. Also called standard deviation from regression. See also: Regression analysis.

## Standard Normal Distribution

A normal distribution (p. 41) with a mean (p. 37) of zero and a standard deviation (p. 64) of one. It is in terms of this distribution that tables of areas under the normal curve are stated. Thus, to use these tables for purposes such as statistical inference (see below) it is necessary to "standardize" the distribution. In simple terms, this is done by converting the point or interval of interest from units into numbers of standard deviations from the mean. After doing this the tables can be used to determine the percentage of the total distribution which is included in the area of interest.

## Statistic

A measure, quantity or value which is calculated from a sample (p. 59) rather than from the population (p. 49).

## Statistical Inference

Using information contained in a sample (p. 59) to make predictions about a larger set, the population (p. 49).

## Statistical Method of Cost Estimating

A method of cost estimating utilizing statistically determined cost estimating relationships (p. 16) which express cost as a function (p. 26) of the characteristics specified for the case in question. A valuable aspect of statistical estimating is that an objective statement regarding cost uncertainty can be provided. See also: Analog method of cost estimating, and Cost estimating relationship.

## Stochastic

A variable (p. 69) or process involving randomness. A variable is stochastic if the value it assumes is governed by chance and the values it may assume can be described by a probability distribution (p. 51).

Stochastic Model
See: Model.

## Suboptimization

Selection of the best alternative course of action which pertains to a subproblem, that is, to only part of the overall problem or objective. Suboptimization is usually necessary because alternatives at all the various levels of decision making cannot, as a practical matter, be analyzed simultaneously before decisions are made at any level. Also referred to as any intermediate stage in a long-run goal attainment program.

## Sunk Costs

Costs which have already been incurred and will not be increased or decreased by any decision made either now or in the future. Therefore such costs have no relevance to decisions regarding future action. For example, in making a decision as to
whether a new plant should be constructed, the construction cost of the existing plant is a sunk cost.

Supply
Defined similarly to demand (p. 18) except from the seller's rather than the consumer's viewpoint.

Systems Analysis
Systems analysis may be viewed as the search for and evaluation of alternatives which are relevant to defined objectives, based on judgment and, wherever useful on quantitative methods, with the objective of presenting such evaluations to decisionmakers for their consideration. It emphasizes the system concept under which any course of action designed to achieve an objective is viewed as a system requiring inputs and producing outputs. The inputs and outputs involved may take on any of a large variety of forms. In this sense, systems analysis encompasses both cost-benefit (p. 15) and costeffectiveness ( p . 16) analyses as well as other types of analysis which may be more limited in scope.
t Distribution

A probability distribution (p. 51) which is used in the same manner as is the normal distribution (p. 41). A distinction is that the $t$ distribution should be used in cases in which the variance (p. 70) of the population (p. 49) is unknown. However, in relatively large samples ( 30 or more) the normal distribution approximates the $t$ distribution to $a$ sufficient degree that the normal distribution is ordinarily used. The major use of the tistribution is for cases in which the sample is of a small size. Also called "Student's t distribution)" because the pen name of the developer (W. S. Gossett, 1876--1937) of the distribution was "Student."

Coefficients (p. 11) obtained from an input-output analysis (p. 29) model (p. 38) which show what fixed percentage input from each industry of the economy is required to produce a unit of output from a specific industry. For example, in a threeindustry economy the output from the steel industry might be made up of .3 of its own output plus .2 of the output of the coal industry plus .1 of the output of the rail transportation industry $+x$ units demanded by final consumers. In this case . $3, .2$, and . 1 would be the technical coefficients.

## Technological Spillover

A spillover (p. 64) which affects the relationship between physical outputs and physical inputs of some external entity which does not pay or receive payment for the spillover. For example, chemical fumes from an industrial plant which reduce (or increase) the yield of crop land. See also: Pecuniary spillover and Spillover.

## Time Series

A display of data showing the magnitude of the same phenomenon at various points in time. An example would be a display of the U.S. population at ten-year intervals from 1900 to 1970.

## Transfer Payments

In economics, grants of money that do not call for any quid pro quo. Examples are: (1) payments from social insurance programs that are not selfsupporting, and (2) veterans' bonuses. In national income accounting, transfer payments are not included in Gross National Product (p. 28) but are included in personal income accounts.

## Transitivity

The characteristic that if a relation between A and $B$ is the same as that between $B$ and $C$ that same relation exists between $A$ and $C$. A relation which possesses this characteristic is equality. Another example is in the consumer theory aspect of economics where an assumption is often made that if a consumer prefers mix of goods and services $A$ to that of mix $B$ and he also prefers mix $B$ to mix $C$ he must prefer mix $A$ to mix $C$.

## Trust Funds

Federal receipts which are earmarked for specific purposes. Two major trust funds are the Federal old age and survivors' insurance (social security) fund and the Federal highway trust fund.

## Uncertainty

In general, uncertainty and risk (p. 58) are used as synonymous terms. A distinction sometimes made between risks and uncertainty is that an event may be risky if a probability distribution (p. 51) can be ascertained. It is uncertain if the probability of success or failure cannot be ascertained.

## Universe

See: Population.

## Utility

In economics, the real or fancied ability of a good or service to satisfy a human want. Usually synonymous with satisfaction, pleasure, or benefit. See also: Marginal utility.

Variable

A quantity that may increase or decrease without other essential changes.

A cost which fluctuates in total in direct proportion to changes in production or operating volume but which remains the same for each unit change in production or operations. See also: Differential cost; Fixed cost; Marginal cost; Sunk costs; Out-of pocket costs.
Variable Input

A factor of production whose quantity is subject to control by the producing unit during the period in question. See also: Marginal cost per unit of input.

Variance
A measure of the degree of spread among a set of values; a measure of the tendency of individual values to vary from the mean (p. 37) value. It is computed by subtracting the mean value from each value, squaring each of these differences, summing these results, and dividing this sum by the number of values in order to obtain the arithmetic mean of these squares. The square root of the variance is the standard deviation (p. 64).

## Vector

A quantity having magnitude and direction. It may be considered to be a matrix (p. 36) of either several columns and one row or several rows and one column. A vector may be contrasted with a scalar (p. 59) which has only magnitude and no direction. It is described by a set of numbers in much the same way as a point on a map is described by its coordinates.

An illustration of subgroupings within a large group or universe. It is used to aid in determining the extent to which individual members of the universe are included in one or more subgroups. In the Venn diagram below the entire area of the rectangle represents a group of people, which is the relevant universe and is labeled $U$. The circle labeled A represents the percentage of persons in the group who are left-handed, while the circle labeled B represents the percentage of women in the group. The shaded area which is called the intersection of $A$ and $B$ thus represents the percentage of lefthanded women in the group.

VENN DIAGRAM


## Welfare Economics

The study of the economic well-being of all persons as consumers and as producers, and possible ways in which that well-being may be improved. It is also known as normative price theory.

## Wholesale Price Index

An index number (p. 29) which is computed monthly by the Bureau of Labor Statistics and which is
designed to show the average change in large-lot prices of commodities sold in primary markets in the United States. The commodities used in the development of the index include, for example, grains on the organized exchanges and machinery f.o.b. manufacturer's plant.

Zero-Sum Game

A game in which the sum of the gains ( $X$ wins two points) exactly equals the sum of the losses ( $Y$ loses two points).

